



## Dr. Lee Dickey's Constructions of the 13-gon

Email Tuesday Nov 11.97

I have prepared four nice pictures for you, based on four constructions that I will list at the end of this message. There is a "base figure" common to all three, the circle inscribed in a square. I have adopted a North, East, South, and West labels for the basic four directions of the circle.

I am trying to create a language for documenting Cabri steps. I want a person (like you, for instance) to be able to read this and turn it into a Cabri construction or even a straightedge and compass construction. My goal is to have it

- (1) crystal clear
- (2) compact

In my stuff that follows, I am trying out my current version of this language. I would be interested in your reaction. Please don't puke.

---

Back to the 13-gon.

I have done three angles, alpha, beta, and gamma, each better than the one before by a factor of 50 to 100. All three use a common "Base Figure". The Base figure has the circle and the square around it. I think that alpha+ appears somewhere in your work, or maybe it is similar to something I thought I saw there. The beta is my first stab at getting  $13 \cdot \alpha$  within tenth of a degree of 360. It is off by about 1/40 of a degree.

The Gamma is an improvement. It is accurate to about 1/2000 of a degree, but it has the disadvantage of being more cumbersome to do.

Lee

-+--+--+--+--+--+--+--+--+--+

### The Base figure

Let O, E be any points.

Z = circle (O, E)

L1 = line (O,E)

{E,W} = meet (L1, Z)

L2 = perp (L1, O).

L3 = perp (L2, N)

L4 = perp (L1, E)

L5 = perp (L2, S)

L6 = perp (L1, W).

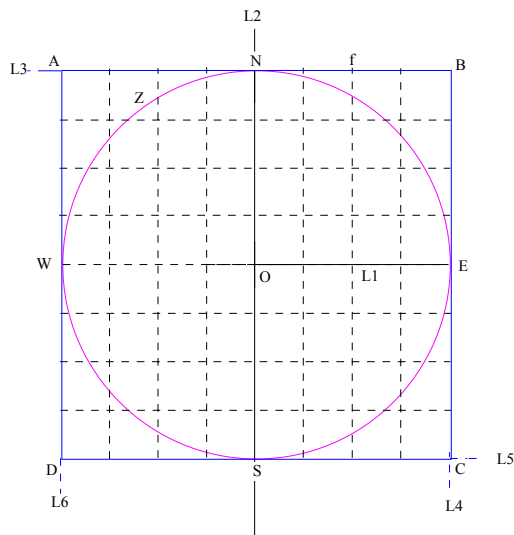
A = meet (L3, L6)

B = meet (L3, L4)

C = meet (L4, L5)

D = meet (L5, L6)

-+--+--+--+--+--+--+--+--+--+



The Base Figure

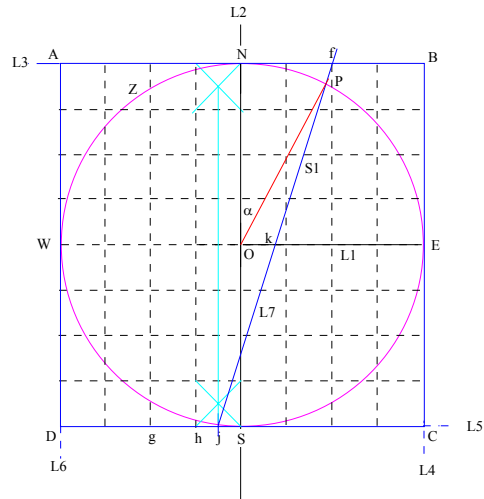
### Construction for alpha

Use the "Base Figure",

- f = mid (N, B)
- g = mid (D, S)
- h = mid (g, S)
- j = mid (h, S)
- L7 = line (f, j)
- k = meet (L1, L7)
- S1 = segment (k, f)
- P = meet (S1, Z)
- alpha = < (N, O, P)

(k is used to define a segment that meets Z in only one point)

alpha = 27.663 504 190 001 236 °  
 13 \* alpha = 359.625 554 470 016 064 °  
 Rel. Error = 0.104 012 647 217 8 %  
 -+--+--+--+--+--+--+--+--+--+



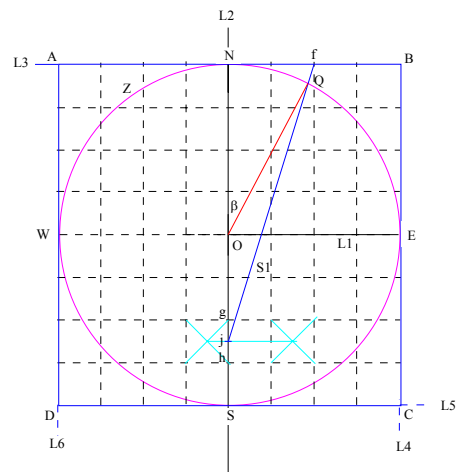
The Base Figure and alpha

### Construction for beta

Use the Base Figure

- f = mid (N, B)
- g = mid (O, S)
- h = mid (g, S).
- j = midpt (g, h).
- S1 = segment (j, f)
- Q = meet (S1, Z)
- beta = < (N, O, Q)

beta = 27.694 119 871 484 227 °  
 13 \* beta = 360.023 558 329 294 945 °  
 Rel. Error = 0.006 543 980 359 7 %  
 -+--+--+--+--+--+--+--+--+--+



The Base Figure and beta

## Construction for gamma

Starting with the Base Figure:

f = mid (N, B)  
 g = mid (D, S)  
 h = mid (g, S)  
 j = mid (h, S)

L7 = bisect ( $\angle$  (E, O, C) )

k = meet (L4, L7)  
 R1 = ray (O, k)  
 m = meet (R1, Z)

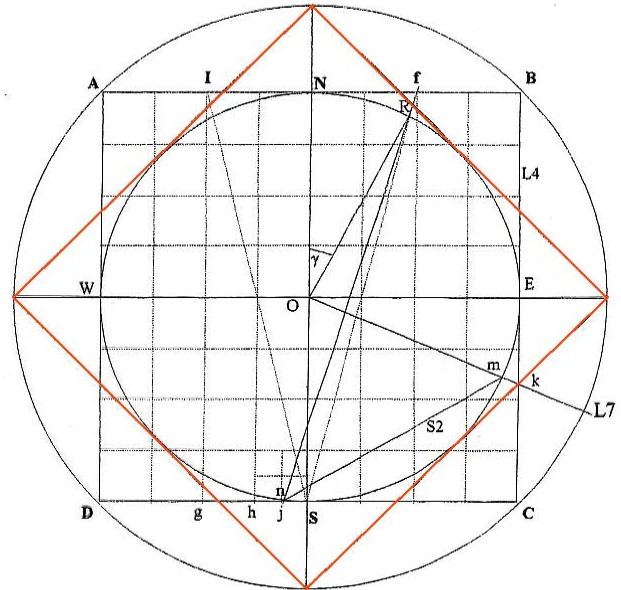
S2 = segment (j, m)  
 n = meet (S2, Z)  
 S3 = segment (n, f)  
 R = meet (S3, Z)  
 gamma =  $\angle$  (N, O, R)

gamma = 27.692 344 071 228 924 ° 13 \*

gamma = 360.000 472 925 976 013°

Rel. Error = 0.000 131 368 326 7 %

-+--+--+--+--+--+--+--+--+



Base Figure and gamma

Vanessa added the second square in red to create the octagon, a geometrical form underlying much of the geometry of Chartres Cathedral. It is possible that the Medieval designers used a 13 pointed star to create the labyrinth, dividing the circle into 13 parts in this way,

**Geometry construction by Lee Dickey, drawings by Vanessa Compton, provocation by Keith Critchlow and Ben Nicholson**

### References:

- Brunés, T. (1967). *The secrets of ancient geometry and its use, Vol. I and II.* (C. M. Napier, Trans.). Copenhagen: Rhodos International Science Publishers.
- Critchlow, K., Jane Carroll, Llewelyn Vaughn Lee. (1973). Chartres maze, model of the universe? *Architectural Association Quarterly*, 5(2), 11–22.
- James, J. (1981) *The contractors of Chartres.* Wyong, Australia: Mandorla Publications
- James, J. (1982). *Chartres, the masons who built a legend.* Boston: Routledge & Kegan Paul.
- Nicholson, B. (1997). *Under Foot and Between the Boards in the Laurentian Library.* The Renaissance Society at the University of Chicago. QV Note 29. Retrieved April 17, 2004, from <http://www.bennicholson.com/ma/maessay.htm>.

### Source:

- Compton, V.J. (2007) *Understanding the Labyrinth as transformative site, symbol and technology: An arts-informed inquiry.* p 221–223